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## Neutrino and antineutrino energy loss rates in massive stars due to isotopes of titanium

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Weak interaction rates on titanium isotopes are important during the late phases of evolution of massive stars. A search was made for key titanium isotopes from available literature and a microscopic calculation of weak rates of these nuclei were performed using the proton-neutron quasiparticle random phase approximation (pn-QRPA) theory. Earlier the author presented the stellar electron capture rates on titanium isotopes. In this paper I present the neutrino and antineutrino energy loss rates due to capture and decay rates on isotopes of titanium in stellar environment. Accurate estimate of neutrino energy loss rates are needed for the study of the late stages of the stellar evolution, in particular for cooling of neutron stars and white dwarfs. The results are also compared against previous calculations. At high stellar temperatures the calculated neutrino and antineutrino energy loss rates are bigger by more than two orders of magnitude as compared to the large scale shell model results and favor stellar cores with lower entropies. This study can prove useful for core-collapse simulators.

### 1. Introduction

The classical work on energy transport by neutrinos and antineutrinos in non-rotating massive stars was performed by Colgate & White <sup>1</sup> and Arnett <sup>2</sup>. Today, despite considerable advancement in the available technology, the collapse simulators find it challenging to generate an explosion out of the collapsing core of massive stars. The prompt shock that follows the bounce of the core stagnates and is not possible to cause a supernova explosion on its own. It loses energy in disintegrating iron nuclei and through neutrino emissions (mainly non-thermal) which are till then transparent to the stellar matter. Various energizing mechanisms for shock revival have been proposed in the text. These include, but are not limited to, the "preheating" mechanism proposed by Haxton <sup>3</sup>, inclusion of magnetic fields (e.g. Ref. <sup>4</sup>) and rotations (e.g. Ref. <sup>5</sup>) in the simulation codes. However to date there have been no successfully simulated spherically symmetric explosions. Even the 2D simulations (addition of convection) performed with a Boltzmann solver for the neutrino transport fails to convert the collapse into an explosion <sup>6</sup>.

Neutrinos radiate around 10% of the rest mass converting the star to a neutron star. Initially the nascent neutron star is a hot thermal bath of dense nuclear matter,  $e^-e^+$  pairs, photons and neutrinos. Neutrinos, having the weak interaction, are most effective in cooling and diffuse outward within a few seconds, and eventually escape with about 99% of the released gravitational energy. Despite the small neutrino-nucleus cross sections, the neutrinos flux generated by the cooling of a neutron star can produce a number of nuclear transmutations as it passes the onion-like structured envelope surrounding the neutron star. Neutrinos from core-collapse supernovae are unique messengers of the microphysics of supernovae. They provide information regarding the neutronization due to electron capture, the infall phase, the formation and propagation of the shock wave and the cooling phase. Cooling rate is one of the crucial parameters that strongly affect the stellar evolution. During the late stages of stellar evolution a star mainly loses energy through neutrinos. White dwarfs and supernovae (which are the endpoints for stars of varying masses) have both cooling rates largely dominated by neutrino production. A cooling proto-neutron star emits about  $3 \times 10^{53}$  erg in neutrinos, with the energy roughly equipartitioned among all three species. The neutrinos and antineutrinos produced as a result of nuclear reactions are transparent to the stellar matter at presupernova densities and therefore assist in cooling the core to a lower entropy state. This scenario does not necessarily hold at extremely high densities and temperatures (this would be the case for stellar collapse where dynamical time scales become shorter than the neutrino transport time scales) where neutrinos can become trapped in the so-called neutrinospheres mainly due to elastic scattering with nuclei. Prior to stellar collapse one requires an accurate determination of neutrino energy loss rates in order to perform a careful study of the final branches of star evolutionary tracks.

The neutrino and antineutrino energy loss rates can occur through four different weak-interaction mediated channels: electron and positron emissions, and, continuum electron and positron captures. The stellar neutrinos are produced due to electron captures and positron decays whereas the antineutrinos are produced due to beta decays and positron captures. As discussed above these neutrinos (antineutrinos) are transparent at presupernova densities and escape the site and help the core achieve a lower entropy which is of vital importance for the core to explode later.

Nabi and Klapdor-Kleingrothaus<sup>7</sup> first reported the calculation of weak interaction rates for 709 nuclei with  $A = 18$  to 100 in stellar environment using the pn-QRPA theory. The authors then presented a detailed calculation of stellar weak interaction rates over a wide range of temperature and density scale for fp/fpg-shell nuclei<sup>8</sup>. Because of the high temperatures prevailing during the presupernova and supernova phase of a massive star, there is a reasonable probability of occupation of parent excited states and the total weak interaction rates have a finite contribution from these excited states. The pn-QRPA theory allows a microscopic state-by-state

calculation of *all* these partial rates and this feature of the model greatly enhances the reliability of the calculated rates in stellar matter. The pn-QRPA model can handle any arbitrarily heavy system of nucleons as it has access to a luxurious model space of up to  $7\hbar\omega$  shells. The pn-QRPA model was successfully used to calculate weak interaction rates on important iron-regime nuclei (e.g. Refs. <sup>9,10,11,12</sup>).

Weak interaction rates on hundreds of nuclei are involved in the complex dynamics of stellar evolution culminating in a supernova explosion. A search for key weak interaction nuclei in presupernova evolution was performed by Aufderheide and collaborators <sup>13</sup>. Late phases of evolution (namely after core silicon burning) in massive stars were considered and a search was performed for the most important electron captures and  $\beta$ -decay nuclei in these scenarios. The lists consisted of dozens of iron-regime nuclei. From these lists electron captures on <sup>49,51,52,53,54</sup>Ti and  $\beta$ -decay of <sup>51,52,53,54,55,56</sup>Ti were short-listed to be of astrophysical importance. Previously Nabi and collaborators <sup>14</sup> presented a detailed analysis of the calculation of stellar electron capture rates on twenty two titanium isotopes. Out of these twenty two isotopes of titanium seven isotopes, namely <sup>49,51,52,53,54,55,56</sup>Ti, are suggested to be important in cooling the core of massive stars through the (anti)neutrino produced via the weak interaction reactions. In this paper I present the neutrino and antineutrino energy loss rates due to these seven isotopes of titanium in stellar matter. The next section discusses briefly the formalism and presents the calculated neutrino and antineutrino energy loss rates. Comparison with previous calculations is also presented in this section. Section 3 finally summarizes the main conclusions.

## 2. Calculations and Results

The Hamiltonian of the pn-QRPA model and its diagonalization was discussed earlier in Ref. <sup>14</sup>. As mentioned in the previous section the neutrino and antineutrino energy loss rates can occur through four different weak-interaction mediated channels: electron and positron emissions, and, continuum electron and positron captures. It is assumed that the neutrinos and antineutrinos produced as a result of these reactions are transparent to the stellar matter during the presupernova evolutionary phases and contributes effectively in cooling the system. The neutrino and antineutrino energy loss rates were calculated using the relation

$$\lambda_{ij}^{\nu(\bar{\nu})} = \left( \frac{\ln 2}{D} \right) [f_{ij}^{\nu}(T, \rho, E_f)] [B(F)_{ij} + (g_A/g_V)^2 B(GT)_{ij}]. \quad (1)$$

The value of D was taken to be 6295s <sup>15</sup>.  $B'_{ij}$ s are the sum of reduced transition probabilities of the Fermi B(F) and Gamow-Teller (GT) transitions B(GT). The effective ratio of axial and vector coupling constants,  $(g_A/g_V)$  was taken to be -1.254 <sup>16</sup>. The  $f_{ij}^{\nu}$  are the phase space integrals and are functions of stellar temperature ( $T$ ), density ( $\rho$ ) and Fermi energy ( $E_f$ ) of the electrons. They are explicitly given

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by

$$f_{ij}^\nu = \int_1^{w_m} w \sqrt{w^2 - 1} (w_m - w)^3 F(\pm Z, w) (1 - G_\mp) dw, \quad (2)$$

and by

$$f_{ij}^\nu = \int_{w_l}^\infty w \sqrt{w^2 - 1} (w_m + w)^3 F(\pm Z, w) G_\mp dw. \quad (3)$$

In Eqs. (2) and (3)  $w$  is the total energy of the electron including its rest mass,  $w_l$  is the total capture threshold energy (rest+kinetic) for positron (or electron) capture.  $F(\pm Z, w)$  are the Fermi functions and were calculated according to the procedure adopted by Gove and Martin<sup>17</sup>.  $G_\pm$  is the Fermi-Dirac distribution function for positrons (electrons).

$$G_+ = \left[ \exp \left( \frac{E + 2 + E_f}{kT} \right) + 1 \right]^{-1}, \quad (4)$$

$$G_- = \left[ \exp \left( \frac{E - E_f}{kT} \right) + 1 \right]^{-1}, \quad (5)$$

here  $E$  is the kinetic energy of the electrons and  $k$  is the Boltzmann constant.

For the decay (capture) channel Eq. (2) (Eq. (3)) was used for the calculation of phase space integrals. Upper (lower) signs were used for the case of electron (positron) emissions in Eq. (2). Similarly upper (lower) signs were used for the case of continuum electron (positron) captures in Eq. (3). Details of the calculation of reduced transition probabilities can be found in Ref.<sup>8</sup>. Construction of parent and daughter excited states and calculation of transition amplitudes between these states can be seen in Ref.<sup>18</sup>.

The total neutrino energy loss rate per unit time per nucleus is given by

$$\lambda^\nu = \sum_{ij} P_i \lambda_{ij}^\nu, \quad (6)$$

where  $\lambda_{ij}^\nu$  is the sum of the electron capture and positron decay rates for the transition  $i \rightarrow j$  and  $P_i$  is the probability of occupation of parent excited states which follows the normal Boltzmann distribution.

On the other hand the total antineutrino energy loss rate per unit time per nucleus is given by

$$\lambda^{\bar{\nu}} = \sum_{ij} P_i \lambda_{ij}^{\bar{\nu}}, \quad (7)$$

where  $\lambda_{ij}^{\bar{\nu}}$  is the sum of the positron capture and electron decay rates for the transition  $i \rightarrow j$ .

The summation over all initial and final states was carried out until satisfactory convergence in the rate calculation was achieved. The pn-QRPA theory allows a microscopic state-by-state calculation of both sums present in Eqs. (6) and (7). This feature of the pn-QRPA model greatly increases the reliability of the calculated

Table 1. Neutrino and antineutrino energy loss rates due to  $^{49,51,52,53}\text{Ti}$  for selected densities and temperatures in stellar matter.  $\log\rho Y_e$  has units of  $g/cm^3$ , where  $\rho$  is the baryon density and  $Y_e$  is the ratio of the lepton number to the baryon number. Temperatures ( $T_9$ ) are given in units of  $10^9$  K.  $\lambda_\nu$  ( $\lambda_{\bar{\nu}}$ ) are the total neutrino (antineutrino) energy loss rates as a result of  $\beta^+$  decay and electron capture ( $\beta^-$  decay and positron capture) in units of  $\text{MeVs}^{-1}$ . All calculated rates are tabulated in logarithmic (to base 10) scale. In the table, -100 means that the rate is smaller than  $10^{-100}\text{MeVs}^{-1}$ .

$\log\rho Y_e$	$T_9$	$^{49}\text{Ti}$		$^{51}\text{Ti}$		$^{52}\text{Ti}$		$^{53}\text{Ti}$	
		$\lambda_\nu$	$\lambda_{\bar{\nu}}$	$\lambda_\nu$	$\lambda_{\bar{\nu}}$	$\lambda_\nu$	$\lambda_{\bar{\nu}}$	$\lambda_\nu$	$\lambda_{\bar{\nu}}$
1.0	0.01	-100	-100	-100	-2.607	-100	-3.449	-100	-1.804
1.0	0.10	-100	-59.070	-100	-2.637	-100	-3.449	-100	-1.804
1.0	0.20	-61.794	-32.015	-100	-2.677	-100	-3.449	-100	-1.802
1.0	0.40	-35.769	-17.468	-91.759	-2.708	-100	-3.449	-100	-1.763
1.0	0.70	-23.720	-10.992	-55.415	-2.723	-73.852	-3.449	-71.037	-1.691
1.0	1.00	-17.711	-9.082	-39.656	-2.724	-52.450	-3.448	-50.164	-1.645
1.0	1.50	-12.765	-7.354	-27.028	-2.686	-35.482	-3.431	-33.669	-1.604
1.0	2.00	-10.130	-6.295	-20.455	-2.596	-26.795	-3.337	-25.268	-1.575
1.0	3.00	-7.258	-4.942	-13.560	-2.356	-17.768	-2.791	-16.639	-1.336
1.0	5.00	-4.550	-3.355	-7.634	-1.667	-9.957	-1.597	-9.321	-0.320
1.0	10.00	-1.609	-1.099	-2.483	-0.095	-3.249	0.239	-2.939	0.904
1.0	30.00	2.887	2.888	3.065	3.660	2.699	3.560	3.149	4.068
4.0	0.01	-100	-100	-100	-2.608	-100	-3.452	-100	-1.805
4.0	0.10	-100	-62.245	-100	-2.638	-100	-3.451	-100	-1.805
4.0	0.20	-58.738	-35.071	-100	-2.678	-100	-3.451	-100	-1.802
4.0	0.40	-32.754	-20.483	-88.744	-2.708	-100	-3.450	-100	-1.764
4.0	0.70	-21.352	-13.359	-53.047	-2.723	-71.484	-3.450	-68.669	-1.691
4.0	1.00	-16.661	-10.131	-38.606	-2.725	-51.400	-3.450	-49.114	-1.645
4.0	1.50	-12.607	-7.511	-26.870	-2.687	-35.324	-3.437	-33.511	-1.605
4.0	2.00	-10.094	-6.329	-20.420	-2.597	-26.759	-3.345	-25.232	-1.575
4.0	3.00	-7.251	-4.947	-13.554	-2.356	-17.762	-2.795	-16.633	-1.336
4.0	5.00	-4.548	-3.355	-7.632	-1.668	-9.956	-1.598	-9.319	-0.320
4.0	10.00	-1.608	-1.098	-2.482	-0.094	-3.247	0.239	-2.938	0.904
4.0	30.00	2.889	2.890	3.067	3.662	2.700	3.562	3.151	4.070
7.0	0.01	-100	-100	-100	-2.939	-100	-3.907	-100	-1.895
7.0	0.10	-74.334	-87.805	-100	-2.960	-100	-3.906	-100	-1.895
7.0	0.20	-40.507	-46.899	-100	-2.990	-100	-3.905	-100	-1.893
7.0	0.40	-22.993	-26.402	-78.982	-3.011	-100	-3.900	-100	-1.850
7.0	0.70	-15.052	-17.539	-46.746	-3.016	-65.184	-3.888	-62.368	-1.770
7.0	1.00	-11.663	-13.895	-33.608	-3.006	-46.402	-3.871	-44.116	-1.719
7.0	1.50	-8.825	-10.799	-23.089	-2.936	-31.543	-3.833	-29.729	-1.673
7.0	2.00	-7.273	-8.936	-17.598	-2.805	-23.939	-3.792	-22.411	-1.640
7.0	3.00	-5.541	-6.528	-11.844	-2.504	-16.052	-3.665	-14.923	-1.391
7.0	5.00	-3.846	-3.945	-6.929	-1.764	-9.253	-2.224	-8.616	-0.360
7.0	10.00	-1.510	-1.189	-2.383	-0.161	-3.149	0.167	-2.839	0.867
7.0	30.00	2.892	2.887	3.070	3.659	2.704	3.559	3.155	4.067
10.0	0.01	2.448	-100	0.705	-100	-2.099	-100	-2.444	-100
10.0	0.10	2.451	-100	0.646	-100	-2.093	-100	-2.440	-100
10.0	0.20	2.452	-100	0.569	-100	-2.092	-100	-2.439	-100
10.0	0.40	2.459	-100	0.503	-100	-2.087	-100	-2.431	-75.486
10.0	0.70	2.481	-85.387	0.474	-63.156	-2.074	-66.149	-2.412	-44.389
10.0	1.00	2.504	-60.612	0.540	-44.980	-2.051	-46.814	-2.393	-31.731
10.0	1.50	2.540	-41.152	0.910	-30.618	-1.973	-31.527	-2.359	-21.662
10.0	2.00	2.573	-31.298	1.281	-23.292	-1.827	-23.724	-2.284	-16.478
10.0	3.00	2.626	-21.274	1.733	-15.781	-1.364	-15.715	-1.280	-11.090
10.0	5.00	2.709	-13.005	2.144	-9.521	-0.132	-9.017	0.447	-6.483
10.0	10.00	2.981	-6.274	2.679	-4.460	1.876	-3.579	2.224	-2.607
10.0	30.00	4.388	1.354	4.582	2.129	4.213	2.038	4.666	2.544
11.0	0.01	5.525	-100	4.591	-100	4.801	-100	4.328	-100
11.0	0.10	5.522	-100	4.578	-100	4.799	-100	4.328	-100
11.0	0.20	5.524	-100	4.558	-100	4.801	-100	4.328	-100
11.0	0.40	5.525	-100	4.547	-100	4.801	-100	4.328	-100
11.0	0.70	5.529	-100	4.542	-100	4.801	-100	4.328	-100
11.0	1.00	5.532	-100	4.547	-100	4.801	-100	4.328	-96.327
11.0	1.50	5.538	-84.221	4.601	-73.687	4.801	-74.596	4.328	-64.731
11.0	2.00	5.543	-63.605	4.705	-55.599	4.801	-56.031	4.329	-48.785
11.0	3.00	5.551	-42.822	4.921	-37.329	4.797	-37.263	4.334	-32.638
11.0	5.00	5.562	-25.953	5.172	-22.468	4.774	-21.964	4.391	-19.431
11.0	10.00	5.630	-12.792	5.455	-10.979	4.928	-10.095	5.057	-9.125
11.0	30.00	6.228	-0.973	6.478	-0.198	6.093	-0.287	6.556	0.219

Table 2. Same as Table 1 but for  $^{54}\text{Ti}$ ,  $^{55}\text{Ti}$  and  $^{56}\text{Ti}$ 

$\log p Y_e$	$T_9$	$^{54}\text{Ti}$		$^{55}\text{Ti}$		$^{56}\text{Ti}$	
		$\lambda_\nu$	$\lambda_{\bar{\nu}}$	$\lambda_\nu$	$\lambda_{\bar{\nu}}$	$\lambda_\nu$	$\lambda_{\bar{\nu}}$
1.0	0.01	-100	-0.174	-100	0.296	-100	1.531
1.0	0.10	-100	-0.174	-100	0.296	-100	1.531
1.0	0.20	-100	-0.174	-100	0.296	-100	1.531
1.0	0.40	-100	-0.174	-100	0.298	-100	1.531
1.0	0.70	-89.296	-0.174	-89.048	0.351	-100	1.531
1.0	1.00	-62.883	-0.174	-62.506	0.488	-74.004	1.531
1.0	1.50	-41.968	-0.174	-41.587	0.710	-49.274	1.531
1.0	2.00	-31.322	-0.174	-30.966	0.854	-36.724	1.531
1.0	3.00	-20.418	-0.169	-20.108	1.017	-23.904	1.531
1.0	5.00	-11.255	-0.081	-11.019	1.196	-13.197	1.542
1.0	10.00	-3.584	1.051	-3.470	1.542	-4.418	2.002
1.0	30.00	2.850	4.052	3.061	4.239	2.552	4.387
4.0	0.01	-100	-0.175	-100	0.296	-100	1.531
4.0	0.10	-100	-0.175	-100	0.296	-100	1.531
4.0	0.20	-100	-0.175	-100	0.296	-100	1.531
4.0	0.40	-100	-0.175	-100	0.298	-100	1.531
4.0	0.70	-86.929	-0.175	-86.680	0.351	-100	1.531
4.0	1.00	-61.833	-0.175	-61.456	0.488	-72.954	1.531
4.0	1.50	-41.810	-0.175	-41.429	0.710	-49.116	1.531
4.0	2.00	-31.286	-0.174	-30.930	0.854	-36.689	1.531
4.0	3.00	-20.411	-0.169	-20.102	1.017	-23.898	1.531
4.0	5.00	-11.254	-0.081	-11.017	1.196	-13.195	1.542
4.0	10.00	-3.583	1.051	-3.469	1.542	-4.417	2.002
4.0	30.00	2.852	4.054	3.062	4.240	2.553	4.389
7.0	0.01	-100	-0.289	-100	0.240	-100	1.498
7.0	0.10	-100	-0.289	-100	0.240	-100	1.499
7.0	0.20	-100	-0.289	-100	0.240	-100	1.499
7.0	0.40	-100	-0.288	-100	0.242	-100	1.499
7.0	0.70	-80.628	-0.287	-80.379	0.301	-96.738	1.499
7.0	1.00	-56.835	-0.285	-56.458	0.448	-67.956	1.500
7.0	1.50	-38.028	-0.280	-37.648	0.680	-45.334	1.500
7.0	2.00	-28.465	-0.274	-28.109	0.828	-33.868	1.502
7.0	3.00	-18.701	-0.261	-18.392	0.996	-22.188	1.504
7.0	5.00	-10.551	-0.201	-10.314	1.179	-12.492	1.516
7.0	10.00	-3.484	0.999	-3.370	1.517	-4.318	1.978
7.0	30.00	2.855	4.050	3.066	4.237	2.557	4.386
10.0	0.01	-100	-100	-100	-100	-100	-100
10.0	0.10	-48.507	-100	-51.341	-100	-100	-100
10.0	0.20	-27.864	-100	-28.499	-88.596	-85.726	-94.766
10.0	0.40	-16.932	-86.965	-16.466	-46.252	-45.055	-49.332
10.0	0.70	-11.544	-50.916	-10.838	-27.654	-27.000	-29.408
10.0	1.00	-8.808	-36.237	-8.361	-19.996	-19.407	-21.185
10.0	1.50	-6.314	-24.554	-6.228	-13.836	-13.225	-14.510
10.0	2.00	-4.890	-18.528	-5.028	-10.624	-9.953	-10.979
10.0	3.00	-3.214	-12.247	-3.614	-7.232	-6.418	-7.185
10.0	5.00	-1.171	-6.834	-1.259	-4.244	-3.020	-3.740
10.0	10.00	1.589	-2.215	1.679	-1.564	0.758	-0.549
10.0	30.00	4.367	2.541	4.577	2.727	4.068	2.917
11.0	0.01	4.577	-100	3.963	-100	3.961	-100
11.0	0.10	4.580	-100	3.963	-100	3.959	-100
11.0	0.20	4.574	-100	3.963	-100	3.958	-100
11.0	0.40	4.577	-100	3.963	-100	3.959	-100
11.0	0.70	4.578	-100	3.962	-100	3.960	-100
11.0	1.00	4.578	-100	3.961	-84.583	3.960	-85.621
11.0	1.50	4.579	-67.600	3.957	-56.898	3.961	-57.458
11.0	2.00	4.580	-50.818	3.955	-42.926	3.963	-43.194
11.0	3.00	4.583	-33.783	3.955	-28.776	3.967	-28.667
11.0	5.00	4.593	-19.774	3.972	-17.189	3.982	-16.635
11.0	10.00	4.815	-8.724	4.556	-8.076	4.261	-7.014
11.0	30.00	6.252	0.219	6.453	0.404	5.964	0.611

rates over other models in stellar matter where there exists a finite probability of occupation of excited states.

The calculated neutrino and antineutrino energy loss rates due to  $^{49,51,52,53}\text{Ti}$  are presented in Table 1 whereas Table 2 presents the corresponding rates due to  $^{54,55,56}\text{Ti}$ . The calculated rates are tabulated on an abbreviated density scale. The first column gives  $\log(\rho Y_e)$  in units of  $gcm^{-3}$ , where  $\rho$  is the baryon density and  $Y_e$  is the ratio of the electron number to the baryon number. Stellar temperatures ( $T_9$ ) are stated in  $10^9 K$ .  $\lambda_{\bar{\nu}}$  ( $\lambda_{\nu}$ ) are the neutrino(antineutrino) energy loss rates in units of  $MeV.s^{-1}$ . The calculated energy loss rates are tabulated in logarithmic (to base 10) scale. In the table, -100 means that the rate is smaller than  $10^{-100} MeV.s^{-1}$ . It can be seen from Table 1 that at low densities and temperatures the antineutrino energy loss rates due to  $^{49,51,52,53}\text{Ti}$  dominate by order of magnitudes and hence more important for the collapse simulators. As  $T_9[K] \sim 30$ , the neutrino energy loss rates try to catch up with the antineutrino energy loss rates. At high stellar densities the story reverses with neutrino energy loss rates assuming the role of the dominant partner. At low densities the antineutrino energy loss rates have a dominant contribution from the positron captures on  $^{49}\text{Ti}$ . As temperature rises or density lowers (the degeneracy parameter is negative for positrons), more and more high-energy positrons are created leading in turn to higher positron capture rates and consequently higher antineutrino energy loss rates. For the remaining isotopes of titanium considered in this study the antineutrino energy loss rates are dominated by the  $\beta$ -decay of these isotopes, except when the stellar core attains high temperature ( $T_9[K] \sim 30$ ). The energy losses shown by  $^{51,52,53,54,55,56}\text{Ti}$  follow a similar trend. At low densities the antineutrino energy loss rates dominate and as the stellar core stiffens to high densities, the neutrino energy loss rates become more important for the collapse simulators. The neutrino and antineutrino energy loss rates increases monotonically with increasing stellar temperatures. From these tables it can be seen that, e.g. at  $\rho Y_e[gcm^{-3}] = 10$  and  $T_9[K] = 30$ , the neutrino and antineutrino energy loss per unit time per  $^{51}\text{Ti}$  nucleus is  $1161.4 MeV.s^{-1}$  and  $4570.9 MeV.s^{-1}$ , respectively. The complete electronic version (ASCII files) of these rates may be requested from the author.

The calculation of neutrino and antineutrino energy loss rates was also compared with previous calculations. For the sake of comparison I took into consideration the pioneer calculation performed by FFN<sup>19</sup> and those performed using the large-scale shell model (LSSM)<sup>20</sup>. The FFN rates were used in many simulation codes (e.g. KEPLER stellar evolution code<sup>21</sup>) while LSSM rates were employed in recent simulation of presupernova evolution of massive stars in the mass range  $11-40 M_{\odot}$ <sup>22</sup>. Figure 1 depicts the comparison of neutrino energy loss rates due to  $^{49}\text{Ti}$  with earlier calculations. Electron captures on  $^{49}\text{Ti}$  during the core silicon burning phases of massive stars are important due to the study performed by Aufderheide and collaborators<sup>13</sup>. As such it is expected that the stellar neutrinos produced as a result of electron capture on  $^{49}\text{Ti}$  may contribute effectively in cooling the stellar core. The upper panel displays the ratio of calculated rates to the LSSM rates,

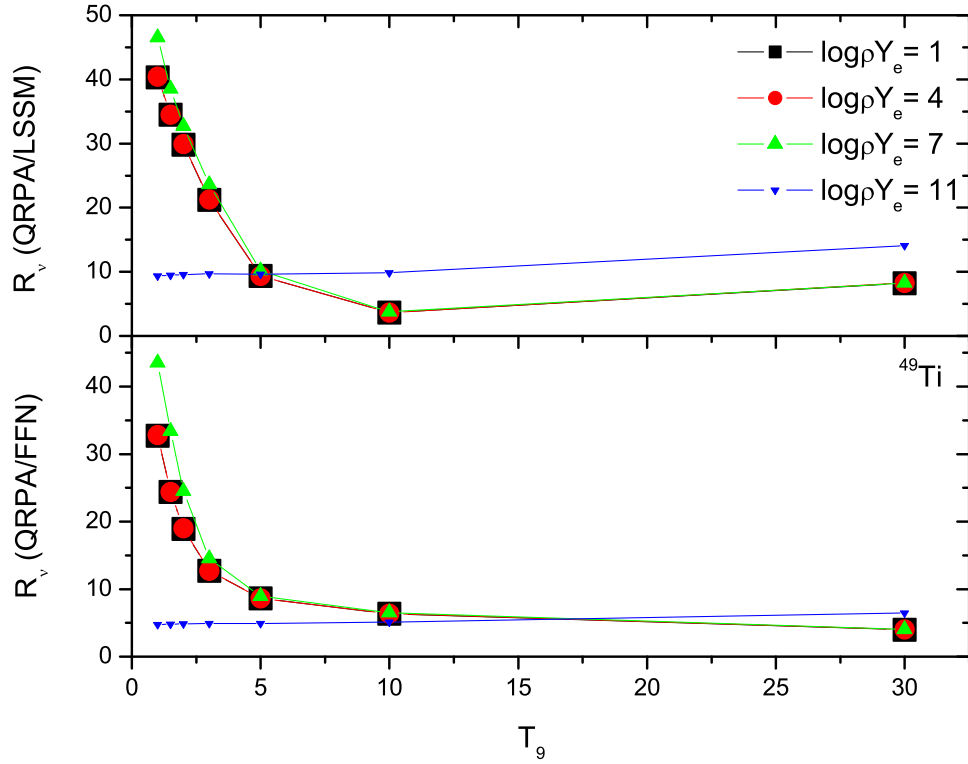


Fig. 1. (Color online) Ratios of pn-QRPA neutrino energy loss rates due to  $^{49}\text{Ti}$  to those calculated using LSSM (upper panel) and by FFN (lower panel) as function of stellar temperatures and densities.  $T_9$  gives the stellar temperature in units of  $10^9$  K. In the legend,  $\log \rho Y_e$  gives the log to base 10 of stellar density in units of  $\text{gcm}^{-3}$ .

$R_\nu(\text{QRPA}/\text{LSSM})$ , while the lower panel shows a similar comparison with the FFN calculation,  $R_\nu(\text{QRPA}/\text{FFN})$ . All graphs are drawn at four selected values of stellar densities ( $\rho Y_e [\text{gcm}^{-3}] = 10^1, 10^4, 10^7$  and  $10^{11}$ ). These values correspond roughly to low, medium-low, medium-high and high stellar densities, respectively. The selected values for temperature on the abscissa are  $T_9[K] = 1, 1.5, 2, 3, 5, 10$  and 30. It can be seen from Figure 1 that the pn-QRPA calculated neutrino cooling rates are bigger than the corresponding LSSM rates by as much as a factor of 50. At  $\rho Y_e [\text{gcm}^{-3}] = 10^1, 10^4, 10^7$  the reported rates are bigger by at least a factor of 40 at  $T_9[K] = 1$ . At  $T_9[K] = 10$  the two rates are in reasonable comparison



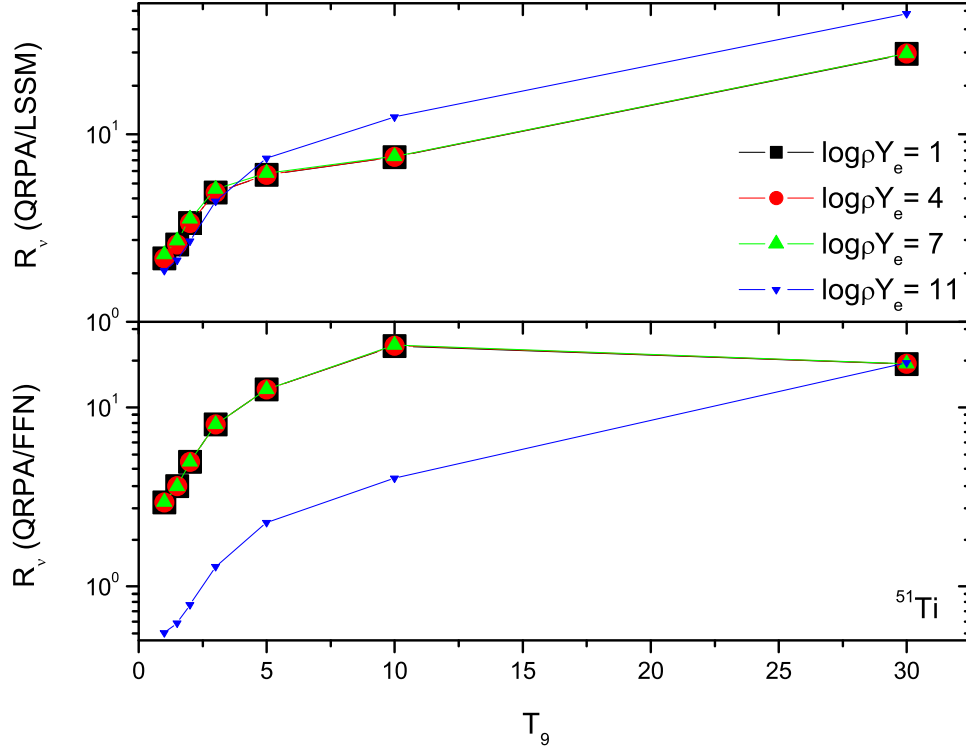


Fig. 2. (Color online) Same as Figure 1 but for neutrino energy loss rates due to  $^{51}\text{Ti}$ .

(pn-QRPA rates are still bigger by a factor of 4). At high densities the pn-QRPA rates are bigger roughly by an order of magnitude. The pn-QRPA rates are also bigger than FFN rates (lower panel) by as much as a factor of 44 at  $T_9[K] = 7$  and  $\rho Y_e [gcm^{-3}] = 10^7$ . At high temperatures and densities the rates are in reasonable comparison (within a factor of 5).

For the case of  $^{51}\text{Ti}$  the pn-QRPA and LSSM rates are in much better agreement (Figure 2). The upper panel shows that till  $T_9[K] \sim 5$  the rates are within a factor of five (with pn-QRPA rates exceeding the LSSM rates). At higher temperatures the pn-QRPA rates surpass the LSSM by as much as a factor of 44. A similar comparison is shown in the lower panel of Figure 2 where the pn-QRPA neutrino energy loss rates are bigger by as much as a factor of 22 compared to FFN rates.

FFN did not calculate the neutrino energy loss rates due to  $^{52,53,54}\text{Ti}$  whereas

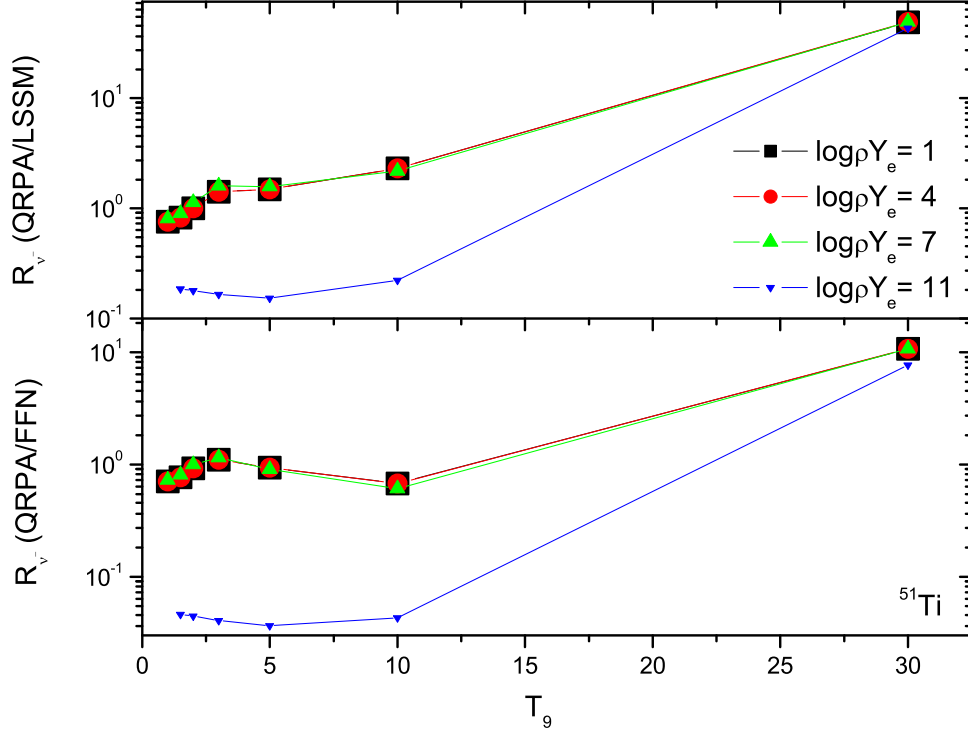


Fig. 3. (Color online) Same as Figure 1 but for antineutrino energy loss rates due to  $^{51}\text{Ti}$ .

LSSM did not calculate the neutrino energy loss rates due to  $^{53,54}\text{Ti}$  and as such a mutual comparison with these previous calculations was not possible for the case of  $^{52,53,54}\text{Ti}$ .

Next I move to the comparison of the antineutrino energy loss rates with previous calculations. Here five cases, namely  $^{51,52,53,55,56}\text{Ti}$ , were possible for mutual comparison with LSSM and FFN rates (FFN did not calculate the antineutrino energy loss rates due to  $^{54}\text{Ti}$ ).

Figure 3 shows the comparison with LSSM and FFN rates for the case of  $^{51}\text{Ti}$ . Here one sees that the LSSM and pn-QRPA rates are in very good comparison at  $\rho Y_e [\text{gcm}^{-3}] = 10^1, 10^4, 10^7$  (within a factor of 2) except at  $T_9 [\text{K}] = 30$  where the reported rates are bigger by around a factor of 50. At high densities the LSSM rates are bigger roughly by a factor of 5. At higher temperatures,  $T_9 [\text{K}] \sim 30$ , the pn-QRPA rates are again bigger by a factor of 43. The lower panel shows that the reported antineutrino energy loss rates are in very good comparison with the FFN

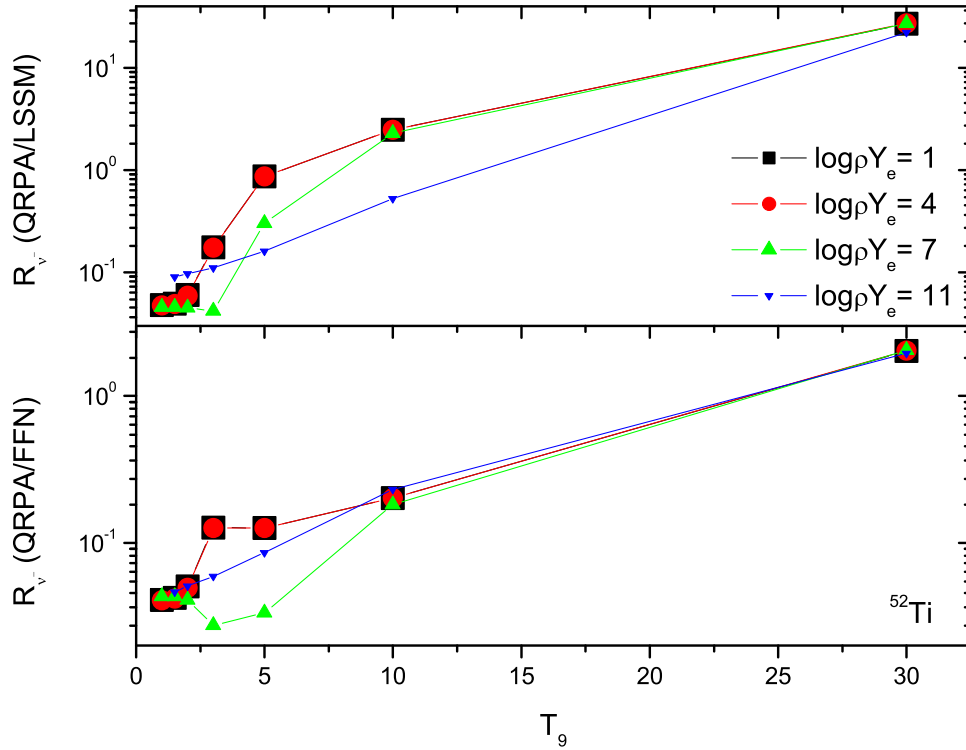


Fig. 4. (Color online) Same as Figure 1 but for antineutrino energy loss rates due to  $^{52}\text{Ti}$ .

rates at  $\rho Y_e [\text{gcm}^{-3}] = 10^1, 10^4, 10^7$  with a perfect comparison at  $T_9[K] = 3$ . At higher temperatures ( $T_9[K] \sim 30$ ) the pn-QRPA rates are bigger by a factor of 10. Only at high densities the FFN rates are bigger (by as much as a factor of 25). However at  $T_9[K] = 30$  the reported rates are again bigger by a factor of 7.

The comparison of antineutrino energy loss rates for the case of  $^{52}\text{Ti}$  is depicted in Figure 4. Here one notes that at low temperatures the LSSM rates are bigger by as much as a factor of 20. The comparison improves as the stellar temperature increases. Within the temperature range  $5 \leq T_9[K] \leq 10$  the comparison is fairly good. The pn-QRPA rates keep enhancing as the temperature of the stellar core increases. Finally at  $T_9[K] = 30$  the pn-QRPA rates are bigger by as much as a factor of 27. At higher temperatures excited state GT strength distributions are required for the calculation of weak rates (parent excited states have a finite prob-

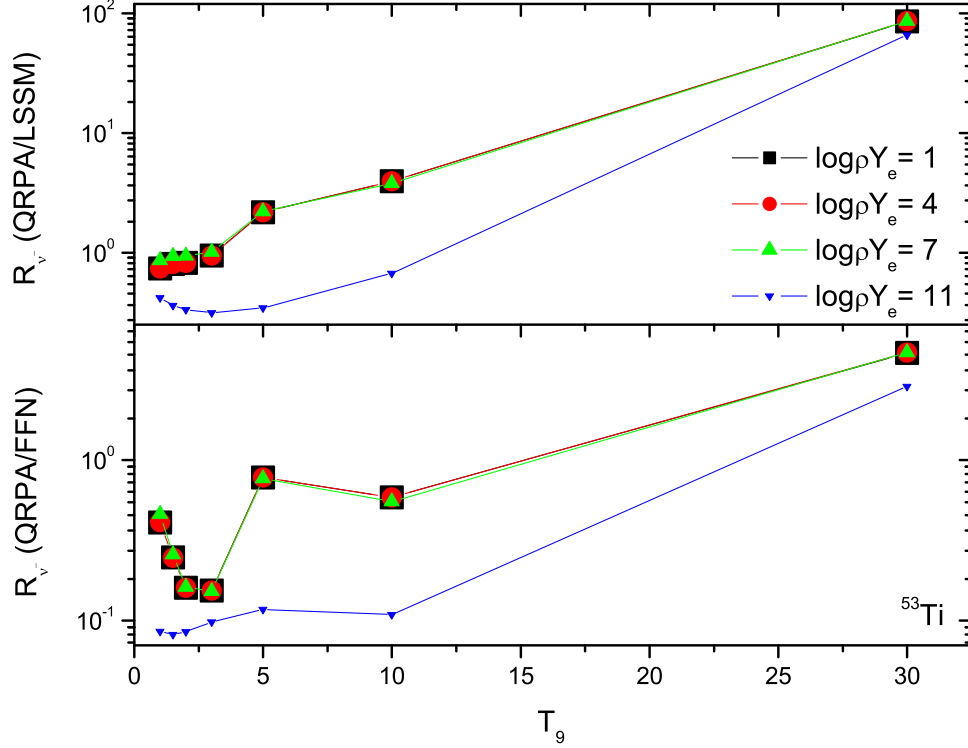


Fig. 5. (Color online) Same as Figure 1 but for antineutrino energy loss rates due to  $^{53}\text{Ti}$ .

ability of occupation). The LSSM employed the so-called Brink's hypothesis in the electron capture direction and back-resonances in the  $\beta$ -decay direction to approximate the contributions from high-lying excited state GT strength distributions. Brink's hypothesis states that GT strength distribution on excited states is *identical* to that from ground state, shifted *only* by the excitation energy of the state. GT back resonances are the states reached by the strong GT transitions in the inverse process (electron capture) built on ground and excited states. On the other hand the pn-QRPA model performs a microscopic calculation of the GT strength distributions for *all* parent excited states and provides a fairly reliable estimate of the total stellar rates. A similar comparison is observed against the FFN rates in the lower panel of Figure 4. The FFN rates are bigger by as much as a factor of 25 at low temperatures. The comparison improves as the stellar temperature increases

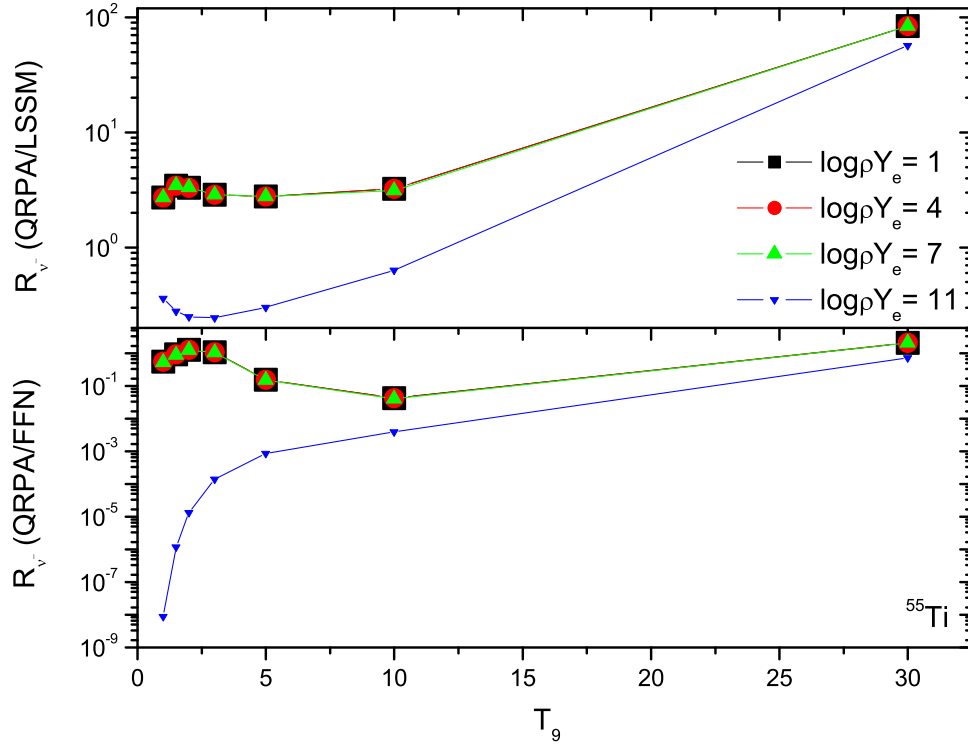


Fig. 6. (Color online) Same as Figure 1 but for antineutrino energy loss rates due to  $^{55}\text{Ti}$ .

and is very good at  $T_9[K] = 30$  (within a factor of 2).

The pn-QRPA antineutrino energy loss rates due to  $^{53}\text{Ti}$  are in good comparison with the corresponding LSSM rates (within a factor of 3) as can be seen from Figure 5. However at  $T_9[K] = 30$  the reported rates are bigger roughly by two orders of magnitude for reasons mentioned before. FFN rates are bigger except at  $T_9[K] = 30$ .

For the case of  $^{55}\text{Ti}$ , the pn-QRPA and LSSM rates are in good comparison (see Figure 6). Once again the reported rates surpass the LSSM rates roughly by two orders of magnitude at  $T_9[K] = 30$ . Looking at the lower panel one sees a staggering 8 orders of magnitude bigger FFN rates at low temperatures and densities. However for the same temperature and density domain the reported rates are in good agreement with the corresponding LSSM rates hinting towards the fact

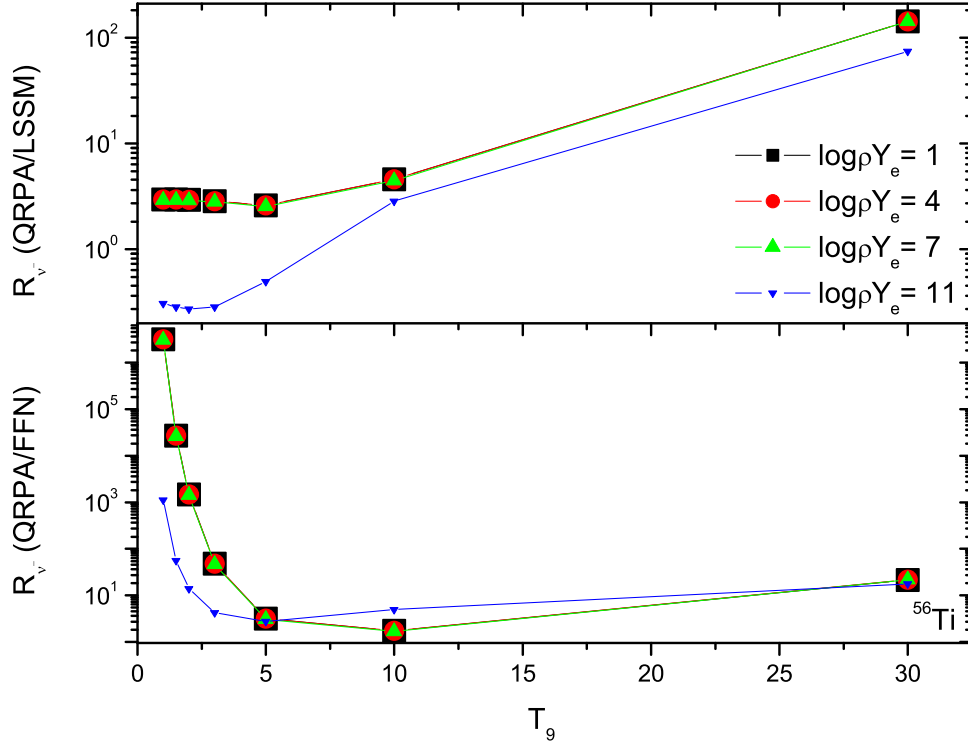


Fig. 7. (Color online) Same as Figure 1 but for antineutrino energy loss rates due to  $^{56}\text{Ti}$ .

that FFN overestimated their antineutrino energy loss rates. It is worth mentioning that these antineutrino energy loss rates are very small numbers ( $\sim 10^{-80}$ ) and can change by orders of magnitude by a mere change of 0.5 MeV, or less, in parent or daughter excitation energies and are more reflective of the uncertainties in the calculation of energies. The comparison is again good at  $T_9[K] = 30$ .

Finally I present the comparison of antineutrino energy loss rates due to  $^{56}\text{Ti}$  with earlier calculations in Figure 7. The upper panel shows the fact that the pn-QRPA rates are in good comparison with the LSSM rates (within a factor of 5) except at  $T_9[K] = 30$  where the reported rates are bigger than two orders of magnitude. The FFN rates are smaller by around 6 orders of magnitude at  $T_9[K] = 1$ . For the same physical conditions the reported rates are in good comparison with the LSSM numbers again hinting towards some problems in the FFN calculations.

Unmeasured matrix elements for allowed transitions were assigned an average value of  $\log ft = 5$  in FFN calculations. On the other hand these transitions were calculated in a microscopic fashion using the pn-QRPA theory (and the large scale shell model) and depict a more realistic picture of the events taking place in stellar environment.

### 3. Conclusions

Isotopes of titanium are amongst the key iron-regime nuclei that play a consequential role in the late phases of stellar evolution of massive stars. The weak-interaction mediated reactions on these nuclei, namely electron capture and  $\beta$ -decay, change the lepton-to-baryon fraction ( $Y_e$ ) during the late phases of stellar evolution. The electron capture contributes in decreasing  $Y_e$  while the  $\beta$ -decay causes an increment in the  $Y_e$  value. The temporal variation of  $Y_e$  within the core of a massive star has a pivotal role to play in the stellar evolution and a fine-tuning of this parameter at various stages of presupernova evolution is the key to generate an explosion. The neutrinos and antineutrinos produced as a result of these weak interaction reactions are transparent to the stellar matter at presupernova densities and therefore assist in cooling the core to a lower entropy state. A lower entropy environment can assist to achieve higher densities for the ensuing collapse generating a stronger bounce and in turn forming a more energetic shock wave. A search was made from the literature to short list seven key titanium isotopes in this respect. The pn-QRPA theory was employed to microscopically calculate the neutrino and antineutrino energy loss rates due to these seven titanium isotopes. The pn-QRPA model has two important advantages as compared to other models. It can handle any arbitrarily heavy system of nucleons since the calculation is performed in a luxurious model space of up to 7 major oscillator shells. Further it is the only available model that can calculate *all* excited state GT strength distributions in a microscopic fashion which greatly increases its utility in stellar calculations.

The neutrino and antineutrino energy loss rates were calculated on a detailed density-temperature grid point and the ASCII files of the rates can be requested from the author. The calculation was also compared with the earlier pioneer calculations performed by FFN and the recent microscopic large scale shell model calculation. The reported neutrino energy loss rates are bigger by as much as a factor of 44 as compared to LSSM rates at high stellar temperatures. The corresponding antineutrino energy loss rates are bigger by more than two orders of magnitude.

The enhanced pn-QRPA energy loss rates favor cooler cores with lower entropies. This may affect the temperature, entropy and the lepton-to-baryon ratio during the hydrostatic phases of stellar evolution which becomes very important going into stellar collapse. The core-collapse simulators are urged to test run the reported stellar neutrino energy loss rates in core-collapse simulation codes to check for some interesting outcome.

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